Tuning of Liquid-crystal Birefringence using a Square AC Variable Frequency Voltage

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ABSTRACT We demonstrate that the birefringence of the liquid-crystal cell (LCC) can be varied by applying different frequency values of a single applied ac square voltage. For the experimental evaluation of the birefringence, associated with a certain wavelength \( \lambda \), as a function of the frequency \( F_{\text{LCC}} \) of the electrical signal applied to the LCC, we use, for the first time to our knowledge, what we call here a frequency-dependent transmission technique. It consists in measuring the transmission responses between crossed and parallel polarizers as a function of the frequency \( F_{\text{LCC}} \). Experimental tests were carried out using a 7µm-thick E63 nematic liquid-crystal cell and a laser source emitting at \( \lambda=1.55 \) µm with a launch power of -3 dBm. The tuning voltage \( V_{\text{LCC}} \) applied to the LCC is an alternative square wave electrical signal whose frequency ranges from 0.5 kHz to 15 kHz. The peak to peak amplitude of the electrical signal is 5 Volt. The curve of the measured variations of the optical path difference of the LCC versus the frequency \( F_{\text{LCC}} \) has a positive slope. Application to the tuning of the center wavelength of the transmission response of a one stage hybrid birefringent filter is shown as a proof-of-principle test.

Index Terms—birefringence, liquid crystals, birefringent filters, optical communication networks

I. INTRODUCTION

Liquid-crystal cells (LCC) have attracted considerable attention for realizing electrically controllable devices [1-7]. In optical communication networks, they are mainly used to tune the wavelength and transmission bandwidth of hybrid liquid-crystal birefringent filters. This is achieved by varying the birefringence of the LCC when different voltages \( V_{\text{LCC}} \) of a single frequency ac square electrical signal are applied [8-11]. In this paper, we demonstrate that the birefringence of the LCC can also be varied by applying different frequency values \( F_{\text{LCC}} \) of a single applied ac square voltage. We show, as a proof of principle test, the use of such \( F_{\text{LCC}} \) controlled birefringence in the tuning of the center wavelength of the transmission response of a hybrid birefringent plate.

The paper is organized as follows. In Section II, the schematic setup used for the experimental evaluation of the LCC birefringence and the principle of the frequency-dependent transmission technique are presented. In Section III, experimental results of the measured LCC birefringence are given. In Section IV, we demonstrate that this frequency controlled birefringence can be used to tune the center wavelength of the transmission response of a one stage hybrid birefringent filter. Finally, we conclude in Section V.

II. PRINCIPLE OF THE BIREFRINGENCE MEASUREMENT

Figure 1 depicts the schematic setup used for the experimental evaluation of the birefringence, associated with a certain wavelength \( \lambda \), as a function of the frequency \( F_{\text{LCC}} \) of the electrical signal applied to the LCC. The setup is basically an LCC placed between two polarizers. Its optic axis is oriented at the angle + 45° with respect to the transmission axis of the front polarizer.

Figure 1: Schematic Setup for the measure of the birefringence of the liquid-crystal cell as a function of the frequency \( F_{\text{LCC}} \) of a single applied ac square voltage. P : Polariser, A : Analyzer
The technique we use for the evaluation of the birefringence consists in measuring the transmitted intensities between crossed ($I_\perp$) and parallel polarizers ($I_\parallel$) as a function of the frequency $F_{LCC}$. In order to derive the equations governing $I_\perp$ and $I_\parallel$ required to deduce the variation of the birefringence of the LCC as a function of the frequency of the electrical signal applied, let us consider a beam of linearly polarized light emerging from a polarizer and incident normally on to a plane parallel to a lossless birefringent plate of thickness $d$ (Cf. Figure 2). On entering the plate, each ray is divided into two rays with different velocities of propagation, and with their D vectors vibrating in two mutually orthogonal directions at right angles to the direction of the plate normal.

They emerge from the plate with a certain phase difference $\phi$. In figure 2, the plane of drawing is parallel to the plate. $D_e$ and $D_o$ represent the two mutually orthogonal directions of vibrations in the birefringent plate, and OP and OA are the directions that are passed by the polarizer P and the analyzer A respectively. Let $\theta$ be the angle that OP makes with $D_e$ and $\chi$ the angle between OA and OP. The amplitude of the light incident on the plate is represented by the vector $OE$ (parallel to OP); its components in the direction of $D_e$ and $D_o$ are

$$OB = E\cos\theta, \quad OC = E\sin\theta$$

The analyzer A transmits only the components parallel to OA which have amplitudes

$$OF = E\cos\theta \cdot \cos(\theta - \chi), \quad \text{and}$$

$$OG = E\sin\theta \cdot \sin(\theta - \chi)$$

At the output of the plate, the two components differ in phase by the amount $\phi = \frac{2\pi}{\lambda}(n_o - n_e)d$ where $n_o$ and $n_e$ are the ordinary and extraordinary refractive indices of the birefringent plate. The total intensity of the light brought to interference is,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cdot \cos \phi$$

where $I_1 = OF^2, I_2 = OG^2$. Referring to Figure 2, equation (3) becomes [12].

$$I = I_o \left\{ \cos^2 \chi - \sin2\theta \cdot \sin2(\theta - \chi) \cdot \sin^2 \frac{\phi}{2} \right\}$$

where $I_o$ represents the maximum of $I$ for $\chi = \theta = 0$.

We consider two important special cases for $\theta = \pi/4$:

- Polarizer/Analyzer parallel ($\chi = 0$). In this case, equation (4) reduces to

$$I_\parallel = I_o \cos^2 \frac{\phi}{2}$$

(5)

- Polarizer/Analyzer perpendicular ($\chi = \pi/2$). In this case, equation (4) gives

$$I_\perp = I_o \sin^2 \frac{\phi}{2} \cdot$$

(6)

As for the derivation of the LCC birefringence when the voltage-dependent transmission technique is used [11], the accurate method for evaluating $\phi$ is the measure of the ratio $I_\perp/I_\parallel$. Solving equation (5) and (6) with respect of $\phi$ leads to

$$|\phi| = N\pi + 2\tan^{-1} \sqrt{\frac{I_\perp}{I_\parallel}} \quad \text{for N} = 0, 2, 4, 6, \ldots$$

(7)

$$|\phi| = (N + 1)\pi - 2\tan^{-1} \sqrt{\frac{I_\perp}{I_\parallel}} \quad \text{for N} = 1, 3, 5, 7, \ldots$$

(8)
These equations hold for positive \( \eta_\sigma > \eta_\pi \) and negative medium \( \eta_\pi < \eta_\sigma \). After a suitable choice of the integer \( N \), the phase difference \( \varphi \) as a function of the frequency \( F_{\text{LCC}} \) is determined from equations (7) and (8). The optical path difference \( \Delta L_{\text{LCC}} \) (or equivalently the birefringence \( \delta n = \frac{\pi}{\lambda} \frac{\Delta L_{\text{LCC}}}{d} \) can be found straightforwardly from the relation \( \varphi = \frac{2\pi}{\lambda} d \cdot \delta n \).

### III. Experimental Birefringence Measurement Results

As shown in section II, the experimental evaluation of the phase difference \( \varphi \) as a function of the frequency \( F_{\text{LCC}} \) is based on the measure of the transmission responses \( I_L \) and \( I_\parallel \). The experimental tests were carried out using a 7μm-thick E63 nematic LCC. The E63 is nematic from -30°C to +82°C with \( n_e \) (at 589 nm) = 1.742, \( n_\pi \) (at 589 nm) = 0.224 (as measured by Merck), \( \delta n \) (at 1550nm) = 0.2081 (assessed value), \( \Delta \varepsilon = 14.6 \) (measured at 1 kHz and 20°C) and \( \varepsilon_\parallel = 19.5 \). The variation of the dielectric anisotropy \( \Delta \varepsilon \) as a function of the frequency at room temperature is given as [13]:

<table>
<thead>
<tr>
<th>Frequency</th>
<th>100 Hz</th>
<th>1 kHz</th>
<th>10 kHz</th>
<th>100 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \varepsilon )</td>
<td>13.21</td>
<td>12.84</td>
<td>11.94</td>
<td>9.91</td>
</tr>
</tbody>
</table>

Indium-tin-oxide (ITO) thin films were used as transparent electrodes. The optical source is a laser emitting at \( \lambda = 1.55 \) μm with a launch power of -3 dBm. The tuning voltage \( V_{\text{LCC}} \) applied to the LCC is an alternative square wave electrical signal whose frequency ranges from 0.5 kHz to 15 kHz. The peak to peak amplitude of the electrical signal is set to 5 Volt.

![Figure 3: Measured transmitted intensities (I_L) and (I_\parallel) for different values of the frequency of the electrical signal applied to the LCC for \( \lambda = 1.55 \) μm.](image)

Figure 3 shows the transmittance of the liquid crystal cell as a function of the frequency \( F_{\text{LCC}} \) of the applied electrical signal for two orientations of the analyzer A. The dotted-line curve represents the transmitted intensity \( I_L \) when the polarizers are crossed, whereas star-line curve corresponds to the case \( I_\parallel \) where the polarizers are parallel. As can be seen from the figure, two regions can be distinguished:

- For \( F_{\text{LCC}} \leq 5 \) kHz: As the applied frequency decreases, the anisotropy decreases and also the phase shift \( \varphi \). At \( F_{\text{LCC}} = 5 \) kHz, the first maximum of \( I_L \) corresponding to the first minimum of \( I_\parallel \) is matched with a phase shift reaching \( \varphi = \pi \). The liquid crystal cell is then half-waved with the ratio \( \frac{I_L}{I_\parallel} = \infty \). Moreover, the intersection of the two transmissions \( I_L = I_\parallel \) is used to evaluate \( N \) as it corresponds to a quarter-wave liquid crystal cell with \( \varphi = \pi/2 \). The integer \( N \) in the formula (7) is zero \( (N = 0) \) and the phase shift \( \varphi(F_{\text{LCC}}) \) can be calculated for any value of the frequency belonging to this range using equation (7).

- For \( F_{\text{LCC}} > 5 \) kHz: As the applied frequency increases, the birefringence increases and also the phase shift \( \varphi \). Beyond 10 kHz, the birefringence remains constant leading to a constant phase shift \( \varphi \). The integer \( N \) in the formula (8) is one \( (N = 1) \) and the phase shift \( \varphi(F_{\text{LCC}}) \) can be calculated for any value of the frequency in this range using equation (8).

![Figure 4: Measured variation of the optical path difference \( \Delta L_{\text{LCC}} \) as a function of the frequency \( F_{\text{LCC}} \) according to the deduced phase difference \( \varphi \).](image)

Figure 4 shows the variation of the optical path difference \( \Delta L_{\text{LCC}} \) as a function of the frequency \( F_{\text{LCC}} \) for two orientations of the analyzer A. As can be seen, when the frequency \( F_{\text{LCC}} \) of the applied electrical signal varies between 0.5 kHz and 7 kHz, the...
LCC operates in a quasi-linear regime. Then, its birefringence is linear, and its optical path difference $\Delta_{\text{LCC}}$ can be approximated by,

$$\Delta_{\text{LCC}} = \beta \cdot F_{\text{LCC}}$$

(9)

where $\beta = +0.1 \mu m/kHz$ is the positive slope of the curve.

It is useful to recall the variation of the optical path difference $\Delta_{\text{LCC}}$ of the liquid crystal cell when different voltages $V_{\text{LCC}}$ of a single frequency ($F_{\text{LCC}} = 10$ kHz) ac square electrical signal are applied (Conf. Figure 5). For $V_{\text{LCC}}$ below the threshold voltage (3.5 V), $\Delta_{\text{LCC}}$ (and consequently the birefringence) remains at its maximum value whereas when $V_{\text{LCC}}$ increases beyond the threshold voltage, $\Delta_{\text{LCC}}$ decreases.

Figure 5: Measured variation of the optical path difference of the liquid-crystal cell versus the electrical signal applied voltage for $F_{\text{LCC}} = 10$ kHz and $\lambda = 1.55 \mu m$.

IV. APPLICATION TO THE WAVELENGTH TUNING OF A ONE STAGE FILTER

In this section, we show, as an experimental proof-of-principle test, the use of such frequency controlled birefringence to tune the wavelength of the transmission response of a hybrid birefringent plate (HBP) placed between crossed polarizers, acting as a one stage birefringent filter.

The HBP is composed of a Quartz birefringent plate (BP) with a birefringence of $88.71 \times 10^{-4}$ (at $\lambda = 1.55 \mu m$) and a thickness of 4444 $\mu m$ coupled to a 7$\mu m$-thick E63 nematic (E. Merk Chemicals, Germany) LCC (from ENST de Bretagne, France) (Cf. Figure 6). The fast and slow axes of both plates are parallel to each other and their optic axes are oriented, as previously, at the angle $+45^\circ$ with respect to the transmission axis of the front polarizer. The material then acts as a uniaxial crystal. The amplified spontaneous emission from an erbium-doped fiber is used as a continuous broadband light source. The spectral response output is monitored on an optical spectral analyzer.

The hybrid optical path difference $\Delta$ introduced by the HBP is,

$$\Delta = \Delta_{BP} + \Delta_{\text{LCC}}$$

(10)

where $\Delta_{BP}$ and $\Delta_{\text{LCC}}$ are OPDs introduced, by the birefringent plate and the electrooptic LCC, respectively. The LCC is used as the tuning element. The free spectral range (FSR) of the such one stage birefringent filter and its full width at half maximum (FWHM) are given respectively by [14],

$$FSR = \frac{\lambda_i^2}{\Delta}$$

and

$$FWHM = \frac{0.8 \lambda_i^2}{\Delta}$$

where $\lambda_i$ is the center wavelength which satisfies the condition for the HBP to be half-waved, $\Delta = (1 + 2k) \frac{\lambda_i}{2}$

where $k$ is an integer. Table 1 summarizes the values of the opto-geometrical parameters of the frequency controlled HBP. If we assume initially, that the operating wavelength $\lambda_i = \lambda_c$, the incident light beam the polarization of which is parallel to the direction of the.

Table 1. Opto-geometrical parameters of the one stage tunable hybrid birefringent filter in the linear regime

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSR</td>
<td>58 nm</td>
</tr>
<tr>
<td>FWHM</td>
<td>28 nm</td>
</tr>
<tr>
<td>$\Delta_{BP}$</td>
<td>39.42 $\mu m$</td>
</tr>
<tr>
<td>$\Delta_{\text{LCC}}$</td>
<td>Varies as $F_{\text{LCC}}$ varies</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>1.56 $\mu m$</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>1.55 $\mu m$</td>
</tr>
<tr>
<td>$\lambda_{\text{end}}$</td>
<td>1.58 $\mu m$</td>
</tr>
<tr>
<td>Interference order $k$</td>
<td>26</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>+3.8 nm/kHz (linear regime)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>+0.1 $\mu m$/kHz (linear regime)</td>
</tr>
</tbody>
</table>
front polarizer, turns by 90° and, thus, passes through the rear polarizer without attenuation. If we move away from this wavelength, the hybrid stage is no longer half-wave-d and the polarization at the output becomes elliptic. The light is then attenuated by the rear polarizer and this attenuation becomes stronger as we move away from \( \lambda_c \). It follows that tuning the HBP is equivalent to correcting this elliptic polarization so that the transmission of the rear polarizer becomes maximal at \( \lambda_4 \neq \lambda_c \). This wavelength tunability of the maxima of the transmittivity of the filter is achieved by applying the frequency \( F_{LCC} \) to the LCC. This results in the reconfiguration of the birefringence introduced by the HBP in accordance with equation (10). According to equation (9), the tuning range \( \delta \lambda \) is a linear function of the variation \( \delta (\Delta_{LCC}) \) introduced by the LCC such that \( \delta \lambda = \frac{\lambda}{\Delta} \cdot \delta (\Delta_{LCC}) \).

![Figure 7: Measured tuning of the center wavelength of the hybrid birefringent plate under tuning frequency \( F_{LCC} \).](image)

Figure 7 illustrates the measured wavelength tuning of the one-stage filter. When the applied frequency increases from 0.5 kHz and reaches, for instance, the \( F_{LCC} = 3 \) kHz, then \( \Delta_{LCC} = (\Delta_{LCC}) \). The peak of order \((2k+1)\), where \( k \) is an integer moves toward the great wavelengths without variation of the FSR such that \( \lambda_k = \lambda_1 + \alpha \cdot \frac{\lambda}{F_{LCC}} \) where \( \alpha \) is the tuning rate expressed in nm/kHz. Note that the operating frequencies are determined by \( \alpha = \frac{\delta \lambda}{\delta F_{LCC}} = \frac{\beta}{k} \) and consequently the tuning range \( \delta \lambda \) is limited by the electrooptical characteristics of the LCCs. However, when \( F_{LCC} > 7 \) kHz, we are no longer in the linear regime and the peak of order \((2k+1)\) stops moving toward the great wavelengths indicating the tuning end at \( \lambda_{\text{end}} = \lambda_{\text{end}} \).

### V. CONCLUSION

We have shown that the birefringence of the LCC can be varied by applying different frequency values of a single applied ac square voltage. For the experimental evaluation of the birefringence as a function of the frequency of the electrical signal applied to the LCC, we have used, for the first time to our knowledge, a frequency-dependent transmission technique which consists in measuring the transmission responses between crossed and parallel polarizers as a function of the frequency of the electrical signal applied. Experimental tests were carried out using a 7μm-thick E63 nematic liquid-crystal cell. The tuning voltage applied to the LCC is an alternative square wave electrical signal whose frequency ranges from 0.5 kHz to 15 kHz. The peak to peak amplitude of the electrical signal is set to 5 Volt. Measured variation of the optical path difference of the liquid-crystal cell versus the frequency of the electrical signal applied shows a steep slope around 4 kHz with a positive tuning rate of 0.1 μm/kHz. Application to the tuning of the center wavelength of the transmission response of a one-stage hybrid birefringent filter shows that when the frequency increases, the center wavelength moves towards the great wavelengths in agreement with the positive tuning rate found.

### REFERENCES


