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Discussion on the general oscillation startup condition and the Barkhausen criterion

Fan He · Raymond Ribas · Cyril Lahuec · Michel Jézéquel

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Abstract The Barkhausen criterion has been widely used to determine the oscillation startup condition. However, we propose it is only partially correct. The general oscillation condition and the relation between the general oscillation condition and the Barkhausen criterion are reviewed in this paper. The oscillation frequency related to the general oscillation startup condition is also presented.

Keywords Barkhausen criterion · Feedback oscillator · Oscillation startup condition · Conditionally stability · Sinusoidal oscillation

1 Introduction

Recently, several articles have been written showing the Barkhausen criterion as the determination of oscillation startup condition for a harmonic oscillator [1–3]. In those articles, some examples and the application of the Barkhausen criterion are reviewed. However, none of these papers mention the fact that this criterion cannot be used as a general oscillation condition, since it is based on the steady state response of any given system. This is an important limitation of the Barkhausen criterion. In this paper, we show that the general oscillation startup condition of a feedback oscillator is more complicated than assumed by the Barkhausen criterion. The relation between the Barkhausen criterion and the general criterion is shown. Two types of feedback oscillators are analyzed under the general oscillation criterion. Finally, the frequency determination issue of a feedback oscillator with Barkhausen criterion is discussed and some simulation results are given.

F. He (✉) · R. Ribas
Infineon Technologies, Sophia Antipolis, France
e-mail: fan.he@infineon.com

C. Lahuec · M. Jézéquel
Telecom Bretagne, Brest, France

2 General oscillation condition

A free running harmonic or feedback oscillator can be considered as a linear system as long as its amplitude remains unlimited by its linear output's range. In the following examples, the feedback or oscillator system is considered to be a linear system. The oscillation condition, or stability conditions, of a harmonic oscillator depends on its possible transient response and is therefore best determined from a study of the linear differential equation representing it [4].

The classical mathematic solution of linear differential equation is lengthy so not suitable for engineering use. Fortunately, the Laplace transform offers a good engineering solution of linear differential equation and gives the possibility of getting the system's transient response from the steady state equations. It is well known that the steady state equation of electronic circuits can be formulated by means of mesh or nodal analysis [5]. Then the mesh equations can therefore be taken as a tool to analyze the oscillation condition. The same result can be obtained by means of the nodal equations.

A general mesh equation of a feedback circuit has the form:

$$\begin{cases} Z_{11}I_1 + Z_{12}I_2 + \cdots + Z_{1n}I_n = E_1 \\ Z_{21}I_1 + Z_{22}I_2 + \cdots + Z_{2n}I_n = E_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ Z_{n1}I_1 + Z_{n2}I_2 + \cdots + Z_{nn}I_n = E_n \end{cases} \quad (1)$$

where n is the number of meshes in the circuit. Z_{ij} is the self-impedance of each mesh (when $i = j$) or the mutual

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impedance between meshes (when $i \neq j$). E_i is the sum of the voltage generator imposed in each mesh. I_i is the steady state mesh current. To calculate the transient response of a harmonic oscillator, Z_{ij} takes its general differential form:

$$Z_{ij} = L_{ij}\frac{\mathrm{d}}{\mathrm{d}t} + R_{ij} + \frac{1}{C_{ij}}\int_0^t \mathrm{d}t \qquad (2)$$

On substituting Z_{ij} by its general differential form Eq. 2, the general mesh equation of the harmonic oscillator becomes:

$$\begin{cases} L_{11}\frac{\mathrm{d}i_1}{\mathrm{d}t} + R_{11}i_1 + \frac{1}{C_{11}}\int_0^t i_1\mathrm{d}t + \cdots + L_{1n}\frac{\mathrm{d}i_n}{\mathrm{d}t} + R_{1n}i_n + \frac{1}{C_{1n}}\int_0^t i_n\mathrm{d}t = 0 \\ L_{21}\frac{\mathrm{d}i_1}{\mathrm{d}t} + R_{21}i_1 + \frac{1}{C_{21}}\int_0^t i_1\mathrm{d}t + \cdots + L_{2n}\frac{\mathrm{d}i_n}{\mathrm{d}t} + R_{2n}i_n + \frac{1}{C_{2n}}\int_0^t i_n\mathrm{d}t = 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ L_{n1}\frac{\mathrm{d}i_1}{\mathrm{d}t} + R_{n1}i_1 + \frac{1}{C_{n1}}\int_0^t i_1\mathrm{d}t + \cdots + L_{nn}\frac{\mathrm{d}i_n}{\mathrm{d}t} + R_{nn}i_n + \frac{1}{C_{nn}}\int_0^t i_n\mathrm{d}t = 0 \end{cases} \qquad (3)$$

Equation 3 is the differential form equation of the oscillator. In Eq. 3, $E_{i,i=1\ldots n}$ is omitted here since there is no external excitation voltage in a free running oscillator. Also, I_i is replaced by i_i to represent the instantaneous current in circuit.

Let us suppose that the possible transients are exponentials having the general form e^{st}. The individual currents $i_{i,i=1\ldots n}$ can therefore be written as $I_i e^{st}$, where the I_i are steady state value whose magnitudes depend on the original disturbance. In general, the possible transients, represented by s, may exist either as real quantities or as conjugate complex pair $s = \delta \pm j\omega$. On substituting $I_i e^{st}$ in Eq. 3 and dividing out the common factor e^{st}, yields:

$$\begin{cases} \left(L_{11}s + R_{11} + \frac{1}{C_{11}s}\right)I_1 + \cdots + \left(L_{1n}s + R_{1n} + \frac{1}{C_{1n}s}\right)I_n = 0 \\ \left(L_{21}s + R_1 + \frac{1}{C_{21}s}\right)I_1 + \cdots + \left(L_{21n}s + R_{2n} + \frac{1}{C_{2n}s}\right)I_{2n} = 0 \\ \cdots\cdots\cdots\cdots \\ \left(L_{n1}s + R_{n1} + \frac{1}{C_{n1}s}\right)I_1 + \cdots + \left(L_{nn}s + R_{nn} + \frac{1}{C_{nn}s}\right)I_n = 0 \end{cases} \qquad (4)$$

It is obvious that $I_1 = I_2 = \ldots I_n = 0$ is always a solution of Eq. 4. Since there are n equations and n currents I_i expected, then the I_i will be uniquely determined, However, if the transient is to exist physically, some of the I_i must be different than zero. This will be possible if s is chosen so that the n equations represent fewer than n independent conditions on the I_i.

From the modern algebraic theories, the general condition for the n equations to represent fewer than n independent relations is that the determinant Δ of the impedance matrix (4) should be zero. Thus a necessary condition should be imposed for a harmonic oscillator in which the transient response exists is:

$$\Delta = |\mathrm{Z}| = 0 \qquad (5)$$

The general form of Δ is a polynomial in s divided by some power of s. the root of Eq. 5 determines the value of s, which can be complex number. The location of s in the complex plane represents the property of transients' response $I_i e^{st}$. If s lies in the left half of the complex plane, the corresponding physical transient response is a damped sinusoid of the form $e^{-\delta t}\cos(\omega t)$. If s lies in the right half-plane, the transient response has the form $e^{\delta t}\cos(\omega t)$. In both cases, δ is positive number. If s is a purely imaginary number, i.e. $\delta = 0$, then the transient is a sinusoid of constant amplitude $I\cos(\omega t)$.

Hence, from the above description, for a harmonic oscillator, Eq. 5 is a necessary a general oscillation condition, since a sustained oscillation requires the roots of Eq. 5 lying in the right half of complex plane.

In practice, an oscillator needs exponentially increasing amplitude to get the oscillation start up. So the general oscillation condition of a feedback oscillator can be assumed as following:

Theorem 1 *There is at least one root of Eq. 5 lying in the right half of the s plane or a double roots on the imaginary axe.*

3 Relation between the general oscillation condition and the Barkhausen criterion

A feedback oscillator is generally modeled by the system described in Fig. 1. It consists of an amplifier block A, which adds enough energy to the loop in order to keep it oscillating, and a frequency selective block B.

Let A(jω) represent the steady state transfer function of the amplifier block, B(jω) is the frequency selective block, since the Barkhausen criterion is based on the steady state characteristic of circuit.

The loop transfer function of the system shown in Fig. 1 is $T(j\omega) = A(j\omega)B(j\omega)$. The Barkhausen criterion is:

$$\angle T(j\omega) = 0 \Rightarrow \omega = \omega_0 \tag{6}$$

$$|T(j\omega_0)| = 1 \tag{7}$$

where ω_0 is the oscillator's angular frequency. Equations 6 and 7 are often referred to as the phase and gain conditions, respectively. According to the Barkhausen criterion, the oscillation frequency is determined by the phase condition (6) in practice. Equation 7 is often used to derive the oscillation start-up condition [1, 2]:

$$|T(j\omega_0)| > 1 \tag{8}$$

However as it is shown later, Eq. 8 is not a true oscillation start-up condition.

Now, analyzing the relation between the Barkhausen criterion and the general oscillation condition obtained in Sect. 2 shows that the Barkhausen criterion, defined by Eqs. 6 and 7, is only a particular case of Eq. 5, a steady state equivalent condition of Eq. 5. Thus, Eq. 8 does not work in some cases and hence it cannot be used as a general startup condition.

Combining Eqs. 6 and 7 together gives:

$$\left.\begin{array}{l}\angle T(j\omega_0) = 0\\ |T(j\omega_0)| = 1\end{array}\right\} \Leftrightarrow T(j\omega_0) = 1 \tag{9}$$

Here, a problem of the application of the Barkhausen criterion can be identified. The problem comes from the sign of loop gain. In Fig. 2, the sign of feedback is shown explicitly. Thus the system in Fig. 1 is just the case where A and B are non-inverting connected Therefore the Barkhausen criterion has the form shown in Eq. 9.

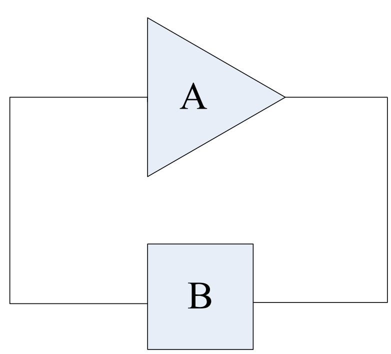

Fig. 1 Generalized feedback oscillator

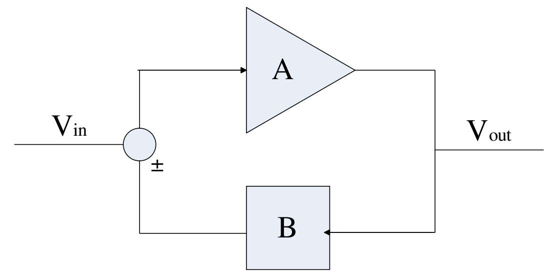

Fig. 2 General feedback system

However, if A and B are connected with a phase shift of π (minus sign), then the Barkhausen criterion should take the following form:

$$T(j\omega_0) = -1 \tag{10}$$

This situation is also explained in [3].

The relation of the Barkhausen criterion and the general oscillation condition can be explained by the feedback network theory.

In the feedback network theory, the system shown in Fig. 2 is studied by means of two fundamental quantities: the feedback factor F and the loop gain T. The transfer gain of the system is well known:

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 \mp AB} = \frac{A}{1 \pm T} = \frac{A}{F} \tag{11}$$

where $T = AB$ is the loop gain and F is the feedback factor.

Applying the general mesh or nodal analysis, the feedback factor is: [5]

$$F = \frac{\Delta}{\Delta_0} \tag{12}$$

where, Δ is the system's principal determinant (impedance or admittance), which is the same as defined in Eq. 5. Δ_0 is the reduced determinant of Δ when the regeneration source component vanishes.

Equation 12 shows that Eq. 5 is equivalent to $F = 0$ ($\Delta_0 \neq 0$), yielding:

$$\Delta = 0 \Leftrightarrow F = 0|_{\Delta_0 \neq 0} \tag{13}$$

It can be found that the Barkhausen criterion Eq. 9 is equivalent to the necessary oscillation condition Eq. 5, when the transient component s is replaced by the steady state component $j\omega$:

$$F = 0|_{\Delta_0 \neq 0} \Leftrightarrow T(j\omega_0) = \pm 1|_{s=j\omega} \tag{14}$$

In the presentation of the Barkhausen criterion, the complex transient component $s = \delta \pm j\omega$ is replaced by a purely imaginary number $j\omega$. This means that the

Barkhausen criterion assumes the physical transient of system has only the steady state form $j\omega$, which corresponds a sinusoidal with constant amplitude. However, this is never true in a real system.

Furthermore, whereas Eq. 8 is widely used as the oscillation startup condition in practice, it has in fact no generality. The steady state loop gain is greater than unity at some frequency it cannot prove there must be a transient that lies in the right half of complex plane.

Nyquist found some feedback circuits, which have $|T(j\omega_0)| > 1$ and $\angle T(j\omega_0) = 0$ at some angular frequency ω_0, that are stable instead of oscillating as predicted by Eq. 8 [6]. This type of feedback circuit is referred to as Nyquist or conditionally stable circuit [6].

The feedback circuit found in [7] has a loop gain:

$$T(s) = \frac{10(s^2 + 2s + 26)}{s(s+1)(s+2)} \tag{15}$$

The steady state form of Eq. 15 is therefore:

$$T(j\omega) = \frac{10(-\omega^2 + 2j\omega + 26)}{j\omega(j\omega + 1)(j\omega + 2)} \tag{16}$$

Applying the Barkhausen criterion (6), it can be found that when $\omega_0 \approx 1.64$ or 4.39, the phase shift of loop gain is zero, while these two angular frequencies also satisfy condition (8). Hence the Barkhausen criterion (6) and (8), lead to the conclusion that the circuit oscillates either at $\omega_0 \approx 1.64$ or $\omega_0 \approx 4.39$.

However applying the general oscillation condition described by Eq. 13, yields:

$$F = 1 + T(s) = 1 + \frac{10(s^2 + 2s + 26)}{s(s+1)(s+2)} = 0 \tag{17}$$

It can be verified by means of the Routh-Hurwitz criterion that the roots of Eq. 17 are all in the left half of complex plane. Therefore, the circuit with the loop gain described in Eq. 15 [7] is in fact a stable circuit, not an oscillator as predicted by the Barkhausen criterion.

4 Analysis of feedback oscillator using the general oscillation condition

To show how to analyze an oscillator with the general oscillation condition, the generalized bridge oscillator shown in Fig. 3 is taken as example.

In Fig. 3, the four impedance elements are named Z_1, Z_2, Z_3 and Z_4 respectively. The operational amplifier OA is modeled as a voltage-controlled voltage source with an output resistance R_0. The input impedance of the OA is assumed to be infinite. The OA's open loop gain is A, which can be frequency dependant, i.e. $A(j\omega)$.

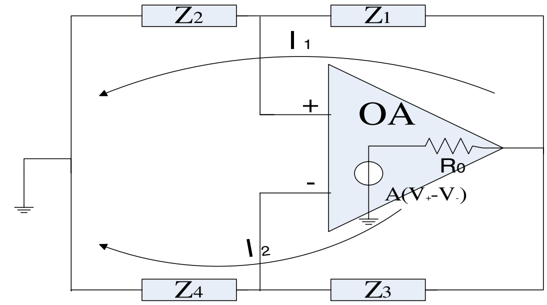

Fig. 3 Generalized Wien bridge oscillator

The mesh equation of the oscillator in Fig. 3 can be found:

$$\begin{bmatrix} Z_1 + Z_2 + R_0 - AZ_2 & R_0 - AZ_4 \\ R_0 - AZ_2 & Z_1 + Z_2 + R_0 - AZ_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{18}$$

By applying condition (5), the necessary oscillation condition is:

$$\Delta = Z_1Z_3 + Z_2Z_3 + Z_1Z_4 + Z_2Z_4 + A(Z_1Z_4 - Z_2Z_3) + R_0(Z_1 + Z_2 + Z_3 + Z_4) = 0 \tag{19}$$

The oscillation occurs only if one of the roots of Eq. 19 lies in the right half of the complex plane. So the real condition can be obtained when Z_1, Z_2, Z_3 and Z_4 are specified and the value of A and R_0 are given. Let us take an example:

$$\begin{cases} Z_1 = R_4 \\ Z_2 = \frac{R_2}{1+sR_2C_2} \\ Z_3 = R_3 \\ Z_4 = \frac{1+sR_1C_1}{sC_1} \end{cases} \tag{20}$$

For simplicity, assuming the amplifier OA is ideal and its output resistance R_0 is zero, Eq. 19 can be simplified as follows:

$$\lim_{A\to\infty} \Delta = s^2R_1R_2C_1C_2 + s\left(R_1C_1 + R_2C_2 - \frac{R_3}{R_4}R_2C_1\right) + 1 = 0 \tag{21}$$

Equation 21 is a quadratic equation of s. The oscillation startup condition therefore is ready to be found by means of the Routh-Hurwitz criterion:

$$R_1C_1 + R_2C_2 - \frac{R_3}{R_4}R_2C_1 < 0 \Leftrightarrow R_3 > R_4\left(\frac{R_1}{R_2} + \frac{C_2}{C_1}\right) \tag{22}$$

More generally, from Eq. 19 it can be concluded that interchanging Z_2 and Z_3 or Z_1 and Z_4 does not change the oscillation startup condition (20).

Therefore, the general oscillation condition described by Eq. 5 offers a systematic way to determine the oscillation condition of a oscillator while avoiding the problem of determination of the feedback sign.

A phase shift oscillator can be taken as another example; Fig. 4 shows a phase shift oscillator with three RC banks.

Assuming the output resistance of the amplifier is zero and the open loop gain is A. The principal determinant of the oscillator can be found:

$$\Delta = \begin{vmatrix} \frac{1}{sC}+R & -R & AR & A(R+R_1) \\ -R & \frac{1}{sC}+2R & -R & 0 \\ 0 & -R & \frac{1}{sC}+2R & R \\ 0 & 0 & R(1+A) & (R+R_1)(1+A)+R_f \end{vmatrix} \quad (23)$$

Again, assuming A very large, Eq. 5 can be calculated as follows:

$$\lim_{A\to\infty} \Delta = 0 \Rightarrow s^3R^3C^3(1+k) + s^2R^2C^2(6+3m) + sRC(5+4m) + (1+m) = 0 \quad (24)$$

where $k = R_f/R_1$, and $m = R/R_1$. Equation 24 is a third order equation of s. The analytical form of the roots is too long to be written here. However, to determine the oscillation condition, it is not necessary to know the exact value of the roots. By applying the Routh-Hurwitz criterion, we have:

Since k and m are both positive, to satisfy the oscillation condition, the third item of the left column of Table 1 should be negative. So the oscillation condition of the phase shift oscillator in Fig. 4 is:

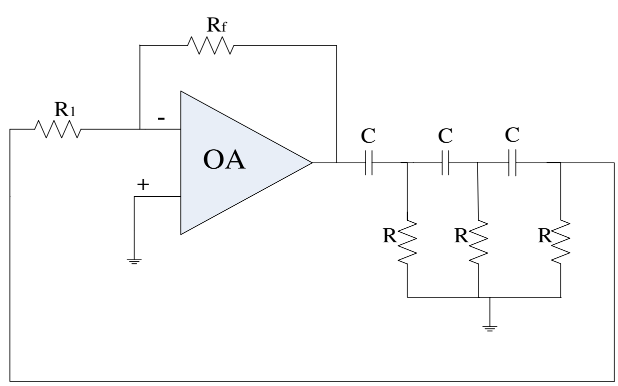

Fig. 4 Phase shift oscillator

Table 1 Routh-Hurwitz criterion table of Eq. 24

n	a_n	a_{n-2}
3	$R^3C^3(1+k)$	$RC(5+4m)$
2	$R^2C^2(6+3m)$	$(1+m)$
1	$RC\left[(5+4m)-\frac{(1+k)(1+m)}{6+3m}\right]$	0
0	$(1+m)$	0

$$(5+4m) - \frac{(1+k)(1+m)}{(6+3m)} < 0 \Rightarrow \quad k > \frac{(5+4m)(6+3m)}{(1+m)} - 1 \quad (25)$$

Hence the oscillation condition is determined by the two factors k and m. Once m is fixed, the value of the feedback ratio k could be determined. For example, if $R_1 \gg R$, thus $m \to 0$, the oscillation condition is $k > 29$.

5 Frequency determination and experiments

Another important subject of harmonic oscillator that has not been discussed in the above section is how to determine the oscillation frequency.

According to the Barkhausen criterion, the oscillator's frequency is calculated from Eq. 6. However this is true only when the transient response has constant amplitude. In practice, the oscillators are always set to have positive. It shows that the actual oscillation frequency is no more the one obtained from Eq. 6.

From the discussion in Sect. 2, the oscillator's output is its physical transient response. Its time domain form is $e^{\delta t}\cos(\omega t)$, thus the oscillation frequency is determined by the imaginary part of s. When the loops gain $T(s)$ changes, the oscillation frequency also changes.

This is better shown using an example. Figure 5 shows the classical Wien bridge oscillator. Let K represent R_3/R_4, then Eq. 21 is reduced to:

$$s^2 + \frac{s(2-K)}{RC} + \frac{1}{(RC)^2} = 0 \quad (26)$$

when $K = 2$, s has two pure imaginary roots $\pm j\omega_0$, where $\omega_0 = 1/(RC)$. This is the same result obtained by means of the steady state equation. However the oscillation frequency can never be ω_0 because the oscillation start-up condition needs $K > 2$. In this case, the roots of quadratic Eq. 26 are:

$$\begin{cases} S_{1,2} = \frac{(K-2)\omega_0 \pm \omega_0\sqrt{(2-K)^2-4}}{2} \\ \omega_0 = \frac{1}{RC} \end{cases} \quad (27)$$

Only when $2 < K < 4$, $s_{1,2}$ is a pair of conjugate complex number. And the actual ω_{act} oscillation frequency when $2 < K < 4$ is determined by the imaginary part of Eq. 27:

$$\omega_{act} = 2\pi f = \frac{\omega_0\sqrt{(4-K)K}}{2} \quad (28)$$

Equation 28 shows that the actual oscillation frequency is not only determined by the value of RC, but also depends on the ratio of two feedback resistors K.

Table 2 Frequency simulation of the Wien bridge oscillator

Loop gain K	Simulation results		Theoretical values		Discrepancy
	Simulated frequency f_{sim} Hz	Normalized $N_{sim} = f/f_0$	$f = \frac{f_0\sqrt{(4-K)K}}{2}$	$N_{cal} = \frac{\sqrt{(4-K)K}}{2}$	$N_{cal} - N_{sim}$
2	35369.1	1	35367.8	1	0
2.01	35369	0.9999972	35367.4	0.999987	−9.673E-06
2.02	35367.8	0.9999632	35366	0.99995	−1.325E-05
2.03	35365.6	0.999901	35363.8	0.999887	−1.355E-05
2.04	35362.6	0.9998162	35360.7	0.9998	−1.624E-05
2.05	35358.6	0.9997031	35356.7	0.999687	−1.568E-05
2.06	35353.8	0.9995674	35351.9	0.99955	−1.752E-05
2.07	35348	0.9994034	35346.1	0.999387	−1.612E-05
2.08	35341.4	0.9992168	35339.5	0.9992	−1.715E-05
2.09	35333.9	0.9990048	35332	0.998987	−1.779E-05
2.1	35325.5	0.9987673	35323.6	0.998749	−1.807E-05
2.2	35191	0.9949645	35190.5	0.994987	2.291E-05
2.3	34968	0.9886596	34967.6	0.988686	2.64E-05
2.4	34656	0.9798383	34653.2	0.979796	−4.244E-05
2.5	34219	0.968529	34244.7	0.968246	−0.0002832

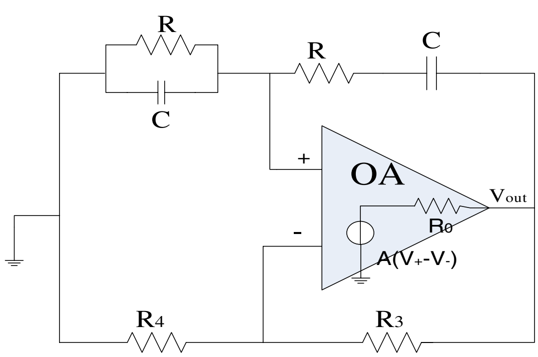

Fig. 5 Classical Wien bridge oscillator

Table 2 shows SPICE simulation results of the oscillator in Fig. 5. To obtain these results, an ideal op-amp (i.e. voltage controlled voltage source, VCVS) is used so that the assumption in Eq. 21 is met. The normalized frequency ω/ω_0 is calculated to show that how far the real frequency differs from the frequency obtained by the Barkhausen criterion. K has been simulated from 2 to 2.5, because above 2.5 the oscillation's amplitude increases so fast that the output's amplitude exceeds the VCVS's amplitude range thus nonlinear amplitude clipping occurs. All the simulated frequency is taken under the linear condition, which means the output's amplitude has no clipping from the amplifier's output range.

It can be shown that the simulated frequency is very close to the frequency calculated using Eq. 28. The discrepancy between the simulation and the calculation has only a difference of 17 ppm (excluding the case when $K = 2.5$, for which it is difficult to get the oscillation frequency without clipping). Hence, Eq. 28 fits well within the linear behaviour of the Wien bridge oscillator.

6 Conclusion

For too long a time, the Barkhausen criterion has been widely used as a condition of oscillation for the harmonic feedback oscillator. It is intuitive and simple to use. However it is wrong, and cannot give the real characteristics of an oscillator such as the startup condition and the oscillation frequency.

The general oscillation condition defined by Eq. 5 and the theorem 1 for the harmonic feedback oscillator is based on the transient response of the system instead of the steady state response used by the Barkhausen criterion. Combining the mesh and nodal equations of circuit, the general oscillation condition offers a systematic and rigorous determination of oscillator's characteristics.

Furthermore the oscillation startup condition (8) related to the Barkhausen criterion is not correct. It is only an intuitive result. Nyquist stability criterion shows that this oscillation startup condition is not true for the conditionally stable circuit.

It also shows that the oscillation frequency determined by the Barkhausen criterion is only the frequency under the ideal conditions. In practice, the oscillator's frequency is not only determined by its filter's parameters but also other circuit parameters such as loop gain, since the transients of a system varies with all the parameters of that system.

For the application, which needs a high frequency precision, the real oscillation frequency should be calculated by a numerical method if the order of the characteristic equation is above 3.

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Fan He was born in Hanzhong, Shaanxi, China, on 13 Feb 1980. He received the M.S. degree in Electrical Engineering from TELECOM Bretagne, France, in 2004. Since 2005 he has been with Infineon Technologies R&D center (Sophia Antipolis, France) where he is currently an analog and mixed-signal design engineer. His research interests are in the feedback network theory, design of low power and high stability oscillators.

Raymond Ribas received the M.S. in 1996 from Bordeaux I university. From 1997 to 2001 I was with ST microelectronics, with Tachys from 2001 to 2002 and after with Infineon up to September 2008. Now he is with Altis Semiconductor.

Cyril Lahuec was born in Orléans, France, in 1972. He received the B.Sc (Hon.) degree from the University of Central Lancashire, Lancashire, U.K., in 1993, the M.Eng and Ph.D. degrees from Cork Institute of Technology, Cork, Ireland, in 1999 and 2002, respectively. He was with Parthus Technologies (now Ceva), Cork, for his Ph.D work and then as a Consultant. He joined the Electronic Engineering Department of Télécom Bretagne, Brest, France, as a Full-Time Lecturer in 2002. His research interests are in frequency synthesis, analogue integrated circuit design, and channel decoding.

Michel Jézéquel (M'02) was born in Saint Renan, France, on February 26, 1960. He received the degree of "Ingénieur" in electronics from the École Nationale Supérieure de l'Électronique et de ses Applications, Paris, France in 1982. In the period 1983–1986 he was a Design Engineer at CIT ALCATEL, Lannion, France. Then, after an experience in a small company, he followed a one year course about software design. In 1988, he joined the École Nationale Supérieure des Télécommunications de Bretagne, where he is currently Professor, head of the Electronics Department. His main research interest is circuit design for digital communications. He focuses his activities in the fields of Turbo codes, adaptation of the turbo principle to iterative correction of intersymbol interference, the design of interleavers and the interaction between modulation and error correcting codes.